Asymptotic Capacity and Optimal Precoding in MIMO Multi-Hop Relay Networks

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ABSTRACT:
A multi-hop relaying system is analyzed where data sent by a multi-antenna source is relayed by successive multi-antenna relays until it reaches a multi-antenna destination. Assuming correlated fading at each hop, each relay receives a faded version of the signal from the previous level, performs linear precoding and retransmits it to the next level. Using free probability theory and assuming that the noise power at relays— but not at destination—is negligible, the closed-form expression of the asymptotic instantaneous end-to-end mutual information is derived as the number of antennas at all levels grows large. The so-obtained deterministic expression is independent from the channel realizations while depending only on channel statistics. This expression is also shown to be equal to the asymptotic average end-to-end mutual information. The singular vectors of the optimal precoding matrices, maximizing the average mutual information with finite number of antennas at all levels, are also obtained. It turns out that these vectors are aligned to the eigenvectors of the channel correlation matrices. Thus they can be determined using only the channel statistics. As the structure of the singular vectors of the optimal precoders is independent from the system size, it is also optimal in the asymptotic regime.

Index Terms—asymptotic capacity, correlated channel, free probability theory, multi-hop relay network, precoding.

I. INTRODUCTION

Many cooperation strategies have been proposed in the literature based on different relaying techniques, such as amplify and forward (AF), decode and forward (DF) and coded cooperation, compress and forward (CF) etc. when these schemes are employed in a pair-wise cooperating system as shown in the below figure. We can assume that, at each instant in time, only one user acts as the source while the other user serves as the relay that forwards the source’s message to the destination. The role between the source and the relay can be interchanged at any instant in time.

Fig.1 COOPERATIVE COMMUNICATION

If the DF scheme is employed the relay will decode and regenerate a new message to the destination in the subsequent time slot. At provide better detection performance. As an extension to the DF scheme, the message generated by the relay can be re-encoded to provide addition error protection, and such a scheme can also be referred to as coded cooperation.

Fig.2 DECODE AND FORWARD REALY

If the AF scheme is employed, the relay simply amplifies the received signal and forwards it directly to the destination without explicitly decoding the message.

Fig.3 AMPLIFY AND FORWARD REALY

The SR scheme, on the other hand, is a dynamic scheme where relays are selected to retransmit the source message only if the relay path is sufficiently reliable. This scheme can be applied on the top of both AF and DF schemes to improve cooperation efficiency. Among the many cooperation schemes proposed in the literature, DF,
AF, and SR schemes are the most basic and widely adopted. More sophisticated schemes, such as the CF scheme can also be devised by exploiting the statistical dependencies between the messages received at the relay and destination but require higher implementation complexity.

This method is perhaps closest to the idea of a traditional relay. In this method a user attempts to detect the partner’s bits and then retransmits the detected bits (Fig. 4). The partners may be assigned mutually by the base station, or via some other technique. For the purposes of this tutorial we consider two users partnering with each other, but in reality the only important factor is that each user has a partner that provides a second (diversity) data path. The easiest way to visualize this is via pairs, but it is also possible to achieve the same effect via other partnership topologies that remove the strict constraint of pairing. Partner assignment is a rich topic whose details are beyond the scope of this introductory article.

2.1.2 DECODE-AND-FORWARD COOPERATIVE

In this scheme, two users are paired to cooperate with each other. Each user has its own spreading code, denoted $c_i(t)$ and $c_d(t)$. The two user’s data bits are denoted $b_i(n)$ where $i=1,2$ are the user indices and $n$ denotes the time index of information bits. Factors $a_{ij}$ denote signal amplitudes, and hence represent power allocation to various parts of the signaling. Each signaling period consists of three bit intervals. Denoting the signal of user 1 $X_1(t)$ and the signal of user 2 $X_2(t)$,

$$X_1(t) = [a_{11}b_1(1)c_1(t), a_{12}b_2(2)c_1(t), a_{13}b_1(2)c_1(t) + a_{14}b_2(2)c_1(t)]$$

$$X_2(t) = [a_{21}b_2(1)c_2(t), a_{22}b_2(2)c_2(t), a_{23}b_1(2)c_2(t) + a_{24}b_2(2)c_2(t)]$$

In other words, in the first and second intervals, each user transmits its own bits. Each user then detects the other user’s second bit (each user’s estimate of the other’s bit is denoted $\hat{b}_2(t)$). In the third interval, both users transmit a linear combination of their own second bit and the partner’s second bit, each multiplied by the appropriate spreading code. The transmit powers for the first, second, and third intervals are variable, and by optimizing the relative transmit powers according to the conditions of the uplink and inter-user channels, this method provides adaptability to channel conditions.
2.1.3 AMPLIFY-AND-FORWARD METHODS

Another simple cooperative signaling is the amplify-and-forward method. Each user in this method receives a noisy version of the signal transmitted by its partner. As the name implies, the user then amplifies and retransmits this noisy version. The base station combines the information sent by the user and partner, and makes a final decision on the transmitted bit (Fig. 4). Although noise is amplified by cooperation, the base station receives two independently faded versions of the signal and can make better decisions on the detection of information.

In amplify-and-forward it is assumed that the base station knows the inter-user channel coefficients to do optimal decoding, so some mechanism of exchanging or estimating this information must be incorporated into any implementation. Another potential challenge is that sampling, amplifying, and retransmitting analog values is technologically nontrivial. Nevertheless, amplify-and-forward is a simple method that lends itself to analysis, and thus has been very useful in furthering our understanding of cooperative communication systems.

2.1.4. PRECODING

Precoding is a generalization of beamforming to support multi-layer transmission in multi-antenna wireless communications. In conventional single-layer beamforming, the same signal is emitted from each of the transmit antennas with appropriate weighting such that the signal power is maximized at the receiver output. When the receiver has multiple antennas, single-layer beamforming cannot simultaneously maximize the signal level at all of the receive antennas. Thus, in order to maximize the throughput in multiple receive antenna systems, multi-layer beamforming is required. In point-to-point systems, precoding means that multiple data streams are emitted from the transmit antennas with independent and appropriate weightings such that the link throughput is maximized at the receiver output. In multi-user MIMO, the data streams are intended for different users (known as SDMA) and some measure of the total throughput (e.g., the sum performance) is maximized. In point-to-point systems, some of the benefits of precoding can be realized without requiring channel state information at the transmitter, while such information is essential to handle the co-user interference in multi-user systems.

Precoding for Point-to-Point MIMO Systems

In point-to-point multiple-input multiple-output (MIMO) systems, a transmitter equipped with multiple antennas communicates with a receiver that has multiple antennas. Most classic precoding results assume narrowband, slowly fading channels, meaning that the channel for a certain period of time can be described by a single channel matrix which does not change faster. In practice, such channels can be achieved, for example, through OFDM. The precoding strategy that maximizes the throughput, called channel capacity, depends on the channel state information available in the system.

Statistical channel state information

If the receiver knows the channel matrix and the transmitter has statistical information, Eigen beamforming is known to achieve the MIMO channel capacity. In this approach, the transmitter emits multiple streams in Eigen directions of the channel statistics. As the actual channel realization is unknown at transmitter, interference will appear between the streams.

Full channel state information

If the channel matrix is completely known, singular value decomposition (SVD) precoding is known to achieve the MIMO channel capacity. In this approach, the channel matrix is diagonalized by taking an SVD and removing the two unitary matrices through pre- and post-multiplication at the transmitter and receiver, respectively. Then, one data stream per singular value can be transmitted (with appropriate power loading) without creating any interference whatsoever.

2.3 ASSUMPTIONS AND SYSTEM MODEL

In this project we first consider a system that consists of large relay networks between the source and the destination with out-of-band conferencing links among the relays. In this module we first develop the basic system at various relay location. This should be done by various types of equations that are explained below.

Transmission between the source and destination should happen in two phase. In the first phase the signal from the source is forwarded to the ‘i’ th relay. And that particular relay forwards its signal to the ‘k’ th relay by using various relaying technique. In this paper we discuss about two important relaying schemes they are Amplify and Forward relaying scheme (AF), and Decode and Forward relaying scheme (DF). Transmission schedule should be explained below.

![Fig.7 Propose System Block Diagram](image)

![Fig.8 TRANSMISSION SCHEDULE](image)
assumptions, the source-to-relay and the conferencing transmissions are scheduled during the same time slot.

In practical systems, it is costly to deploy MN conference links, which is exactly the reason why we propose a p-portion conferencing protocol to limit the percentage of conferencing connections. We will study the impact of p on the tradeoff between the system performance and the system installation cost.

We further define the following channel input-output relationship. In the first hop, the received signal $y_i$ at the i-th relay, $i = 1, 2, \cdots, N$, is given as

$$y_i = \sqrt{P_h}x + n_i$$

where ‘x’ is the signal transmitted by the source, $P_s$ is the transmit power at the source node, $h_i$ is the complex channel gain of the i-th source-to-relay link, which is assumed known to the source, and $n_i$’s are the independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) noise with distribution. The received signal at the $(i+k)$-th relay via the conferencing link is given as

$$y_{i+k} = \sqrt{P_c}f_{i+k}y_i + n_{i+k}$$

Where is the complex link gain, is the CSCG noise with distribution and $P_c$ is the transmit power at the conferencing links. Here the constant coefficient is used to satisfy the average transmit power constraint of the conferencing link. During the second hop, $x_i$ with unit average power is transmitted from i-th relay to the destination, and the received signal $y$ at the destination is given as

$$y = \sum_{i=1}^{N} \sqrt{P_r} g_i x_i + n$$

Where $g_i$ is the complex channel gain of the i-th relay-to-destination link, $P_r$ is the transmit power at each relay, and $n$ is the CSCG noise with distribution.

In this paper, we assume that for the i-th relay, it knows $h_i$, $h_j$, $f_{i,j}$, and $g_i$. Where and $A_i$ is the set of indices corresponding to the relays connected to the i-th relay via the conferencing links. In practice, to obtain $h_j$’s one solution is to let the source send out one symbol pilot, and each relay then forward this pilot to the other relays via the conferencing links. After receiving such forwarded pilot signals, each relay can estimate $h_j$’s, since the conferencing link gains $f_{i,j}$’s are assumed to be constant and known. To obtain $g_i$, we assume that the relay to destination links is reciprocal such that only one pilot signals from the destination is needed.

### 2.3.1 Capacity Upper Bound and Achievable Rates

In this model we first calculate the capacity upper bound and achievable rates of the desired network with two different relaying schemes. And also we prove some capacity-achieving results under special condition.

#### 2.3.1.1 Preliminary Results and Capacity Upper Bound

In this section we calculate some preliminary results and the capacity upper bound. For this reason we have to first consider previous results from different reference paper.

Let be independent random variables, whose means and variances are uniformly and positively bounded, respectively. And is given by

$$E\left(\log\left(1 + \sum_{i=1}^{N} x_i\right)\right) - E\left(\log\left(1 + \sum_{i=1}^{N} E(x_i)\right)\right) \xrightarrow{w.p.1} 0$$

$$E\left(\log\left(1 + \sum_{i=1}^{N} x_i\right)\right) - E\left(\log\left(\sum_{i=1}^{N} E(x_i)\right)\right) \xrightarrow{w.p.1} 0$$

By using the above equation and the classic BC cut-set bound is referred from the previous reference papers we will obtain the following capacity upper bound. And is given by the below equations

$$C_{\text{upper}} \leq \frac{1}{2} \log\left(1 + \frac{P_s}{N_0} \sum_{i=1}^{N} |h_i|^2\right)$$

$$\xrightarrow{w.p.1} \frac{1}{2} \log\left(1 + \frac{P_s}{N_0} \sum_{i=1}^{N} E|h_i|^2\right)$$

Where ‘hi’ represents the channel gain of the entire system, let which is positively bounded then we obtain the below equation.

### 2.3.2 DF and AF Achievable Rate

In this module we calculate the achievable rates for both AF and DF relaying schemes; this should be done by using the equations that are explained below. In this section, the authors showed that the DF rate scales at most on the order of $O(\log(\log(N)))$ without conferencing among the relays, where the source chooses an optimal a subset of relays to decode the source message and let the rest keep silent in the second hop transmission.

In this subsection, we adopt a different scheme to require all the relays to decode the source message and transmit in the second hop. Obviously, compared to the previous scheme, our scheme is not optimal in term of relay...
subset selection, while it is enough to show the improvement of the achievable rate scaling behavior introduced by relay conferencing. Note that both the schemes in and our proposed DF scheme require full channel CSI at the source node. Using the p-portion conferencing strategy, the DF rate scales on the order of $O \left( \log (N) \right)$.

Based on the principle of Maximum Ratio Combining (MRC), received SNR at the relay is the sum of the SNRs in the above equation. Thus for the first hop, the maximum rate supported at the $i$-th relay is given in the below equations:

$$\mu_{DF} = \left[ E(|h_i|^2) + \sum_{k=1}^{M} \frac{P_r|f_{i-k}|^2}{E(|g_i|^2)} + \frac{P_gE(|h_i|^2)}{N_0} \right]$$

Which is positively bounded, thus we have $R_i \sim O(\log(N))$. In the second hop, we assume that all relays transmit simultaneously, and the transmit signal at the $i$-th relay is given by

$$x_i = \frac{1}{\sqrt{E(|g_i|^2)}} g_i^* y$$

Thus, the received signal at the destination is given as,

$$y = \sum_{i=1}^{N} \frac{P_r}{E(|g_i|^2)} |g_i|^2 x + n$$

and the maximum rate supported in the second hop is given as

$$R_{MAC} = \frac{1}{2} \log \left( 1 + \frac{Q_0^2}{N_0} \right)$$

$$\sim \log \left( \frac{Q_0}{\sqrt{N_0}} \right)$$

$$\xrightarrow{w.p.1} \log \left( \frac{E(Q_0)}{N_0} \right)$$

$$= \frac{1}{2} \log \left( \frac{P_r}{N_0} N^2 \mu^2 \right)$$

Where the above equation is valid when $N \to \infty$, and let $E(Q_0)$ is calculated by using the below equation,

$$E(Q_0) = \sqrt{P_r} \sum_{i=1}^{N} \sqrt{E(|g_i|^2)}$$

And $\mu$ factor should be calculated by the below equation

$$\mu = \frac{1}{N} \sum_{i=1}^{N} \sqrt{E(|g_i|^2)}$$

And $R_i$ should be calculated using the below equation

$$R_i = \frac{1}{2} \log \left( 1 + \frac{(M+1)P_r\mu_{DF}}{N_0} \right)$$

After calculating all the parameter’s for the Decode and forward relaying scheme. Next we have to find the achievable rate for the DF scheme this should be done by the below equation,

$$R_{DF} = \min\{\min[R_i], R_{MAC}\}$$

III. AF ACHIEVABLE RATE

In this subsection, we discuss the AF relaying scheme. Since we assume no global CSIs at the relays, the network wide optimal combining at the relays as proposed in cannot be deployed. Thus, with only local CSIs, MRC across conferencing signals is another good choice, which maximizes the received SNR at the relays. Unfortunately, MRC makes the rate expression too complicated to obtain any clean results. Instead, here we combine the received signals $y_i$ and $y_{i-k}$'s at the $i$-th relay as

$$t_i = h_i^* y_i + \sum_{i=1}^{M} \frac{P_r E(|h_{i-k}|^2)}{P_c |f_{i-k}|^2} h_{i-k}^* y_{i-k}$$

Then the transmit signal at the $i$-th relay is given as

$$x_i = a_i \sqrt{P_r} g_i^* t_i$$

Where $a_i$ is the power control factor and it is chosen as by the following equation,

$$a_i^2 = E^{-1}g_i^2 \left[ \sum_{k=0}^{N} |h_{i-k}|^2 + \sum_{k=0}^{N} E( |h_{i-k}|^2 \left( 1 + \frac{P_r E(|h_{i-k}|^2)}{P_c |f_{i-k}|^2} \right) \right]^{-1}$$

This combining scheme is not valid for the case without relay conferencing, i.e., the conferencing link SNR $P_c/N_0 = 0$. Moreover, if $|f_i,i+k|$ or $P_c/N_0$ is close to zero, it will boost the conferencing link noise $n_{i,i+k}$, which may make the performance even worse than the case without conferencing. However, our analysis will show that for uniformly and positively bounded $|f_i,i+k|$'s and arbitrary $P_c/N_0$, the AF scheme performs well as $N \to \infty$.

The received signal at the destination is given as in the above equation. Then, the AF achievable rate is given as

$$R_{AF} = \frac{1}{2} \log \left( 1 + \frac{P_r P_c Q_2^2}{(P_r Q_2 + P_c Q_3 + 1)N_0} \right)$$

Where
Achievable rate of the Amplify and Forward relay scheme should be calculated by using the below equation,

\[
Q_1 = \sum_{i=1}^{N} a_i |g_i|^2 \left( \sum_{k=0}^{M} |h_{i-k}|^2 \right)
\]

\[
Q_2 = \sum_{i=1}^{N} \left( \sum_{k=0}^{M} a_{i+k} |g_{i+k}|^2 \right) |h_i|^2
\]

\[
Q_3 = \sum_{i=1}^{N} \sum_{k=1}^{M} |a_i|^2 \frac{P_o E(|h_{i-k}|)^2 + N_o}{P_c |f_{i-k}|^2} |g_i|^4 |h_{i-k}|^2
\]

As \( N \to +\infty \), we have

\[
E(|h_i|^2)^2 = NM \mu_2
\]

Where

\[
\mu_2 = \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{M} |a_i|^2 \frac{P_o E(|h_{i-k}|)^2 + N_o}{P_c |f_{i-k}|^2} E(|g_i|^4) E(|h_{i-k}|)^2
\]

Since we assume that \( E(|h_i|^2), E(|g_i|^2) \) and \( E(|g_i|^4) \) are uniformly and positively bounded, \(|a_i|\), \(\mu_1\), \(\mu_2\), and \(\mu_3\) are also bounded and positive. For the \( p \)-portion conferencing scheme, since \( E(Q3) \) scales on a smaller order than \( E(Q2) \) as \( N \) goes to infinity, we obtain the AF rate as

\[
R_{AF} = 2 \log \left( \frac{P_o P_{Q1}^2}{N_o} \right) - \log (p_r P_{Q2} + P_{Q3} + 1)
\]

Hence, we obtain

\[
Q_1 = \left( \sum_{i=1}^{N} a_i |g_i|^2 \right) \sum_{k=1}^{M} |h_{i-k}|^2
\]

The term \( Q3 \) is the contribution of the conferencing link noises. Since \( E(Q3) E(Q2) \to 0 \), we conclude that for the \( p \)-portion conferencing scheme, the conferencing link noises are asymptotically negligible as \( N \to +\infty \). This suggests that for large relay networks with AF, we do not need high quality conferencing links, i.e., even with small \( Pc/No \), and the performance of the AF scheme is reasonably good for large \( N \). It is difficult to verify whether the AF scheme is capacity achieving or not for the case with \( 0 < p < 1 \) and generally distributed \( h_i' \)'s and \( g_i' \)'s. In the following, we prove two special capacity-achieving cases, which may be applied to many widely-used scenarios. If \( h_i' \)'s and \( g_i' \)'s are i.i.d., respectively, the AF scheme asymptotically achieves the capacity upper bound as \( N \) goes to infinity for arbitrary \( 0 < p < 1 \) and \( Pc/No > 0 \).

Since \( h_i' \)'s and \( g_i' \)'s are i.i.d, \( E(|h_i|^2), E(|g_i|^2), E(|g_i|^4) \) are identical over different I’s, respectively. Let us examine the term, the power control factor can be rewritten as

\[
a_i^2 \approx \frac{1}{E(|h_i|^2) E(|g_i|^2) E(|g_i|^4)}
\]

For large \( M \), and we have

\[
\frac{E^4(|g_i|^2) N(M + 1)^2}{E^2(|g_i|^2)^2 N M (M + 1)^2 + E(|g_i|^4) MN} \to 1
\]

Hence we have \( \frac{\mu_3}{\mu_2} \to E(|h_i|^2) \). Therefore, the theorem is proved.

For independent but not necessarily identically distributed \( h_i' \)'s or \( g_i' \)'s, the full conferencing scheme, i.e., \( N=M+1 \), asymptotically achieves the capacity upper bound as \( N \) goes to infinity for arbitrary \( Pc/No \); For complete conferencing scheme, we obtain

\[
R_{AF} = \frac{1}{2} \log \left( 1 + \frac{P_o \sum_{k=1}^{N} |h_{i-k}|^2}{(1 + P_{Q2} N_o) N_o} \right)
\]

Therefore, the capacity upper bound is asymptotically achieved.

### SUMRATE CALCULATION FOR VARIOUS CONFERENCING LINK VALUES

In this section we present some simulation results for various values of conferencing value. The conferencing value should be in the range of \( 0 \leq P \leq 1 \). So in this section we have to change the conferencing value between these ranges. And we can check the performance by varying the conferencing link value. And this should be varied by the following way,

\[
\lim_{N \to +\infty} \frac{M + 1}{N} = p
\]
III. SIMULATION RESULTS

In this section we provide some graphical representation of the entire system. In this we calculate the achievable rate and capacity upper bound for the entire system, this should be done by using the above equations. In the below figure, we show the capacity upper bound and the achievable rates for different $p$ values, as the number of relays increases. For the AF relaying scheme, the gap between the upper bound and the achievable rate is very small for $p = 0.2$ and large $N$ values. For the DF relaying scheme, when $N$ is large, we observe that the DF rate and the capacity upper bound have the same scaling behavior.

![Fig.9 Achievable Rates Vs Number Of Relays](image)

Fig.9 Achievable Rates Vs Number Of Relays

Here, we are describing about the achievable rate should be varied by varying the conferencing link value ($p$). By changing the conferencing link value, relay should be used for the transmission should also be changed. The measure of performance used here is achievable rate versus conferencing link value.

![Fig.10 Achievable Rates Vs Conferencing Link Value](image)

Fig.10 Achievable Rates Vs Conferencing Link Value

The same thing we will explain once again here, we calculate the achievable rate by varying the conferencing link signal to noise ratio (snr), the measure of performance used here is achievable rate versus conferencing link signal to noise ratio.

![Fig.11 Achievable Rates Vs Conferencing Link Link Snr](image)

Fig.11 Achievable Rates Vs Conferencing Link Snr

The above figure representing the performance of an large relay network. Here we are changing the mean and the corresponding variance of the channel gain. The measure of performance used here is achievable rate versus number of relays.

![Fig.12 Achievable Rates Vs Number Of Relays](image)

Fig.12 Achievable Rates Vs Number Of Relays

Here we are changing the type of distribution for the channel gain, and check the performance by varying the number of relays present in the system. Here we are using geometric distribution for the creation of channel gain.

![Fig.13 Achievable Rates Vs Number Of Relays](image)

Fig.13 Achievable Rates Vs Number Of Relays
measure of performance used here is achievable rate versus number of relays present in the system.

![Poisson Distribution](image1)

**Fig. 14** Achievable Rates Vs Number Of Relays

Here we are changing the type of distribution for the channel gain, and check the performance by varying the number of relays. Here we are using poisson distribution for the creation of channel gain. The measure of performance used here is achievable rate versus number of relays present in the system.

![Binomial Distribution](image2)

**Fig. 15** Achievable Rates Vs Number Of Relays

Here we are changing the type of distribution for the channel gain, and check the performance by varying the number of relays. Here we are using binomial distribution for the creation of channel gain. The measure of performance used here is achievable rate versus number of relays (n).

**IV. CONCLUSION**

In this system, we investigated the achievable rate scaling laws of the DF and AF relaying schemes in a large Gaussian relay networks with conferencing links. By using this system we also prove the capacity of the system is increased. We showed that for the DF relaying scheme, the rate scales as $O(\log(N))$, compared to $O(\log(\log(N)))$ for the case without conferencing; for the AF relaying scheme, we proved that if the channel fading coefficients $h_i$’s and $g_i$’s are i.i.d., respectively, or $N = M + 1$, it asymptotically achieves the capacity upper bound as $N$ goes to infinity.

**REFERENCE**