Channel Capacity Estimation in MIMO Systems Based on Water-Filling Algorithm

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ABSTRACT

In radio, multiple-input and multiple-output (MIMO), is the use of multiple antennas at both the transmitter and receiver to improve communication performance. Multiple antennas may be used to perform smart antenna functions such as spreading the total transmit power over the antennas to achieve an array gain that incrementally improves the spectral efficiency. In this paper the water filling algorithm has been implemented for allocating the power to the MIMO channels for enhancing the capacity of the MIMO network and is compared with without water filling algorithm.

Keywords: Multi Input Multi Output (MIMO), water filling, Capacity, outage probability, Signal to Noise Ratio (SNR).

1. INTRODUCTION

As it is known that Multiple-Input Multiple-Output (MIMO) systems are used to get higher data rate as compared to a normal SISO system where we keep the same power budget and SNR. A comparison of MIMO system with a SIMO reveals that the MIMO system need lesser transmit power than the SISO system in order to achieve the same capacity. As we need to minimize the energy consumed by the circuit and want to maximize the capacity of a system and that is possible only if we use multiple MIMO system. So a comparative analysis is done to find a system which is more energy efficient.

MIMO system utilizes space multiplex by using antenna array to enhance the efficiency in the used bandwidth. These systems are defined spatial diversity and spatial multiplexing. Spatial diversity is known as Tx -and Rx- diversity. Signal copies are transferred from another antenna, or received at more than one antenna. With spatial multiplexing, the system carriers’ more than one spatial data stream over one frequency, simultaneously.

The article is organized as follows. In section 2, discusses the System Model, Section 3 water filling algorithm. Section 4 we conclude our discussion.
2. SYSTEM MODEL

Diagram of a MIMO wireless transmission system is shown below:

![Diagram of MIMO wireless transmission system]

**Figure 1: MIMO wireless transmission**

The transmitter and receiver are equipped with multiple antenna elements. The transmit stream go through a matrix channel which consists of multiple receive antennas at the receiver. Then the receiver gets the received signal vectors by the multiple receive antennas and decodes the received signal vectors into the original information.

Here is a MIMO system model:

![MIMO system model]

**Figure 2: MIMO system model**

There are detail explains for denoted symbols:

- $r$ is the $M \times 1$ received signal vector as there are $M$ antennas in receiver.
- $H$ represents the channel matrix.
- $s$ is the $N \times 1$ transmitted signal vector as there are $N$ antennas in transmitter.
- $n$ is an $M \times 1$ vector of additive noise term.

Let $Q$ denote the covariance matrix of $x$, then the capacity of the system described by information theory as below:

$$C = \log_2 [\det (I_M + HH^H)] \quad \text{b/s/Hz}$$

This is optimal when is unknown at the transmitter and the input distribution maximizing the mutual information is the Gaussian distribution. With channel feedback may be known at the transmitter and the optimal is not proportional to the identity matrix but is constructed from a water filling argument as discussed later. The form of equation gives rise to two practical questions of key importance. First, what is the effect of $Q$? If we compare the capacity achieved by $Q = (\rho/N)I_n$ and the optimal $Q$ based on perfect channel estimation and feedback, then we can evaluate a maximum capacity gain due to feedback. The second question concerns the
effect of the H matrix. For the i.i.d. Rayleigh fading case we have the impressive linear capacity growth discussed above. For a wider range of channel models including, for example, correlated fading and specular components, we must ask whether this behavior still holds. Below we report a variety of work on the effects of feedback and different channel models.

It is important to note that can be rewritten as:

\[
C_{\text{EF}} = \sum_{i=1}^{m} \log_2 \left(1 + \frac{\rho}{N} \lambda_i \right) \quad \text{b/s/Hz}
\]

Where \( \lambda_1, \lambda_2, \ldots, \lambda_m \) are the nonzero eigenvalues of \( W \), \( m = \min(M,N) \), and

\[
W = \begin{cases} 
HH^*, & M \leq N \\
H^*H, & N < M.
\end{cases}
\]

This formulation can be easily obtained from the direct use of eigenvalue properties. Alternatively, we can decompose the MIMO channel into \( m \) equivalent parallel SISO channels by performing singular value decomposition (SVD) of \( H \). Let the SVD be given by

\[
H = UDV^*
\]

Then \( U \) and \( V \) are unitary and \( D = \text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_1}, \ldots, \sqrt{\lambda_m}, 0, \ldots, 0) \). Hence the MIMO signal model can be rewritten as:

\[
\mathbf{r} = D \mathbf{s} + \mathbf{n}
\]

where \( \mathbf{r} = U^* \mathbf{r}, \mathbf{s} = V^* \mathbf{s} \) and \( \mathbf{n} = U^* \mathbf{n} \).

The above equation represents the system as \( m \) equivalent parallel SISO eigenchannels with signal powers given by the eigenvalues \( \lambda_1, \lambda_2, \ldots, \lambda_m \). Hence, the capacity can be rewritten in terms of the eigenvalues of the sample covariance matrix \( W \). For general \( W \) matrices a wide range of limiting results are known as or both tend to infinity. In the particular case of Wishart matrices, many exact results are also available.

When the channel is known at the transmitter (and at the receiver), then \( H \) is known in above equation and we optimize the capacity over \( Q \) subject to the power constraint \( \text{tr}(Q) \leq \rho \). Fortunately, the optimal \( Q \) in this case is well known and is called a water filling solution. There is a simple algorithm to find the solution and the resulting capacity is given by

\[
C_{\text{WF}} = \sum_{i=1}^{m} \log_2 (\mu \lambda_i)^+ \quad \text{b/s/Hz}
\]
Where $\mu$ is chosen to satisfy

$$\rho = \sum_{i=1}^{m} (\mu - \lambda_i^{-1})^+$$

"+" denotes taking only those terms which are positive. Since $\mu$ is a complicated nonlinear function of $\lambda_1, \lambda_2, \ldots, \lambda_m$, the distribution of $W_{CF}$ appears intractable, even in the Wishart case when the joint distribution of $\lambda_1, \lambda_2, \ldots, \lambda_m$ is known.

If the transmitter has only statistical channel state information, then the ergodic channel capacity will decrease as the signal covariance $Q$ can only be optimized in terms of the average mutual information as

$$C_{\text{statistical-CI}} = \max_Q \mathbb{E} \left[ \log_2 \det \left( I + \rho HQH^H \right) \right]$$

The spatial correlation of the channel has a strong impact on the ergodic channel capacity $C$ with statistical information.

If the transmitter has no channel state information it can select the signal covariance $Q$ to maximize channel capacity under worst-case statistics, which means $Q = (1/N_t)I$ and accordingly.

3. Water filling Algorithm

In this we demonstrated the MIMO channel capacity better than SISO channel capacity and to achieve high capacity gain another method is water filling concept is proposed. In this concept it can also happen that some sub channels that have a poor SNR, do not get any power assigned. Water filling makes sure that energy is not wasted on sub channels that have poor SNR. With water filling, power is allocated preferably to sub channels that have a good SNR. This is optimum from the point of view of theoretical capacity; however, it requires that the transmitter can actually make use of the large capacity on good sub channels.

The basic steps involved in the water filling algorithm is

1. Take the inverse of the channel gains.
2. Water filling has non uniform step structure due to the inverse of the channel gain.
3. Initially take the sum of the total power $P_t$ and the inverse of the channel gain. It gives the complete area in the water filling and inverse power gain.
   $$P_t + \sum_{i=1}^{n} \frac{1}{H_i}$$
4. Decide the initial water level by the formula given below by taking the average power allocated.
5. The power values of each sub channel are calculated by subtracting the inverse channel gain of each channel.

\[
\text{Power}_{\text{allocated}} = \frac{Pt + \sum_{i=1}^{n} \frac{1}{Hi}}{\sum \text{Channel}} - \frac{1}{Hi}
\]

In case the power allocated value become negative stop iteration.

4. SIMULATION RESULTS

Figure 3: Mean Capacity vs SNR

Figure 4: Complementary CDF comparisons (vs capacity) at SNR=10dB

Figure 5: Outage probability comparisons (vs SNR) for Flat Fading Channels
CONCLUSION

This paper we have developed an understanding and described the Mean capacity allocation in a wireless cellular network based on the proposed water filling power allocation in order to enhance the capacity of MIMO systems with different channel assumptions. Here each transmitter decides the distribution of power to the several independent fading channels. We observed Maximum power is allocated to the channel having greater gain. In case of successive power allocation the number of iterations is more here in proposed water filling Algorithm the number of iterations are less. Initial level of the power allocated is close to the ideal value so the results of proposed algorithm are better. Results indicate that the proposed water-filling scheme has better capacity than successive water filling at greater value of power budget. We also discussed the variation of the outage probability of the system.

REFERENCES


